Accounting for Time

- In addition to computing all benefits and costs in money terms,…
- The monetary costs and benefits must be calculated at a single point in time.
- In this way can compare projects that have very different time profiles of benefits and costs.

Compounding

- Interest rate is applied to periodically (every year) – to both principal and earned interest
  - Year 0: 100
    - (x 1.08 – *multiplication factor for 8% interest rate*)
  - Year 1: 108
    - (x 1.08)
  - Year 2: 116.64
    - (x 1.08)
  - Year 3: 125.97
    - etc

Compounding

- Future Value (FV) of X in time period t =

  $X \_ (1+r)^t$

- The value in time t of $1 invested today (time 0).

- Compounding Factor:

  $(1+r)^t$

  (Spreadsheet Example)

Discounting
(Opposite of Compounding)

- Present Value (PV) of = X_t received in time period t =

  $1/(1+r)^t$ _X_n
- Amount of money needed to invest today to obtain X_t in time t
- Discounting Factor

  $(1/(1+r)^t)$

  (Spreadsheet example)

Discounting

- At 8% interest rate,

  $125.97 in year 3 is equivalent to having 100 in year 0.

  - If I invested $100 today, I would have $125.97 three years from now.
  - By discounting we are including the opportunity cost of capital.
    - If the discounted net benefits of a project are positive, the return on this investment are greater than the market rate of interest

Discounting

YR	Costs	Benefits	Net	Discount Factor[a]	PV
1	100	0	-100	.926	-92.59
2	0	109	109	.857	93.45
Total	100	109	9		0.86

[a]Discount at 8%

Discounting

- Discounting permits comparison of projects with different profiles of costs and benefits

Flow of Costs and Benefits

	Option 1			Option 2		
YR	Cost	Benefit	Net Benefit	Cost	Benefit	Net Benefit
0	100	0	-100	40	50	10
1	10	0	-10	40	50	10
2	10	150	140	40	50	10
Sum	120	150	30	120	150	30

Discounting

Option 1

YR	Net Benefits	Compound factor[a]	Discount Factor[a]	PV Net Benefits
0	-100	1.000	1.000	-100
1	-10	1.080	.9259	-9.26
2	140	1.1664	.8573	120.02
Total	30			10.76

[a]Discount at 8%

Discounting
Option 2

YR	Net Benefits	Compound factor[a]	Discount Factor[a]	PV Net Benefits
0	10	1.000	1.000	10.00
1	10	1.080	.9259	9.26
2	10	1.1664	.8573	8.57
Total	30			27.83

[a]Discount at 8%

Issues in Discounting

- Real versus nominal prices and interest rate
- Projects with different time horizons

Real vs Nominal Values

- Inflation rate = i
- $CPI_1 = CPI_0 * (1+i)$
- Real price (in time $_0$ prices) of price X_1 in time $_1$ is:

$$X_1 * (CPI_0/CPI_1)$$

$$= X_1 * (1/(1+i)$$

Real vs Nominal Values

- So if invest $X today at an interest rate of 8% (r = 0.08) and inflation rate is 5% (i = 0.05):

  $FV_{real} = X * (1+r)/(1+i)$

  $= (1.08/1.05) \approx 1.03$

- So, the ***real*** interest rate =

  $(1+r)/(1+i)$

  $\approx (1+(r-i))$

Real vs Nominal Values

- In CBA calculations, may project costs and benefits in nominal or real terms, but need to be consistent with interest rate:
  - If use nominal prices, need to use nominal interest rate
  - If use real prices, need to use real interest rate

Different Time Horizons

- Consider two options:
  1. Hydroelectric dam
     - 75 year life
     - NPV = $30 million
  2. Cogeneration project
     - 15 year life
     - NPV = 24 million
  - *Choose Hydroelectric because higher NPV?*

Different Time Horizons

- Need to put on equal time horizon:
- Equivalent Annual Net Benefit (EANB):
  - NPV / Annuity Factor
  - Gives the amount which, if received every year for the life of the project, would have the same NPV as the project.

Different Time Horizons

- Annuities: receive an equal (nominal) payment every year for a fixed number of years

1. $PV = A(1+r)^{-1} + A(1+r)^{-2} + \ldots A(1+r)^{-t}$
2. $PV(1+r) = A + A(1+r)^{-1} + \ldots A(1+r)^{-(t-1)}$
3. $PV(1+r) - PV = A - A(1+r)^{-t}$ *[2. – 1.]*
4. $PV(1+r-1) = A - A(1+r)^{-t}$
5. $PV = A\,((1-(1+r)^{-t})/r)$

Different Time Horizons

- Annuity factor:
- $((1-(1+r)^{-t})/r)$

This gives the present value of receiving $1 per year for T years.

(Spreadsheet example)

Different Time Horizons

- EANB for Hydroelectric plant
  - $30 million / annuity factor for 75 years (8%)
  - $30 million / 12.4611 = $ 2.407 million
- EANB for cogeneration plant:
  - $24 million / annuity factor for 15 years (8%)
  - $24 million / 8.559 = 2.804 million

Arguments against Discounting

- Particular concern for environmentalists:
  - Discounting undervalues long-term environmental costs of current actions
  - At 5% discount rate, PV of $1 in 30 years is only $0.23
  - “Equal Standing” argument – Future generations should have equal weight

Arguments for discounting in CBA

- Reflects time preferences of consumers
- Opportunity cost of capital
- Without discounting, give too much weight to future generations
  - Assumes that future generations will not benefit from higher incomes (*counter to historical evidence)*
  - The relatively poor current generation has greater weight than the relatively rich later generations